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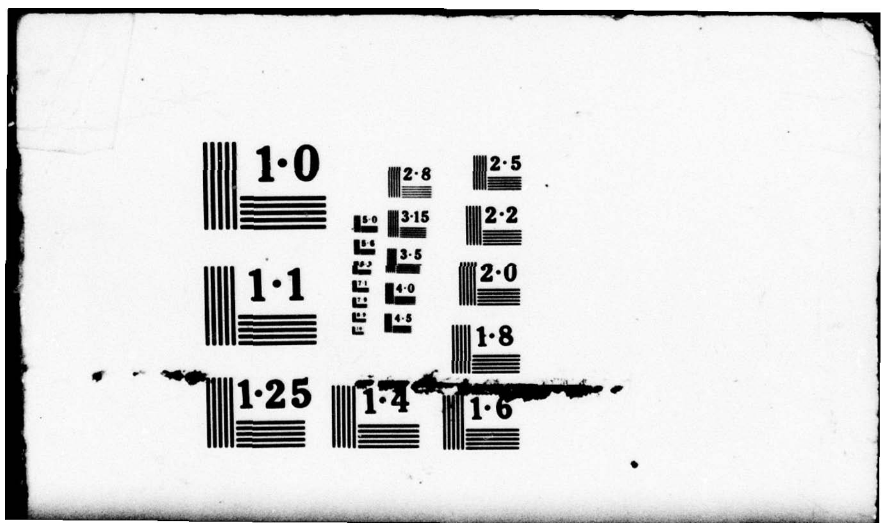
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Texas A&M University
Department of Aerospace Engineering
College Station, Texas 77843

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1 January [illegible] to 31 August 1978

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The results of a two year effort to define and study problems associated with the design of scientific balloons are summarized. These results include: the development of a design program including material deformation; an analysis program for use in off-design configurations; uniaxial and biaxial constant rate testing; uniaxial creep testing; and uniaxial material characterization. In addition, the need for further investigation is elaborated.

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Introduction

In an effort to assist the scientific ballooning community, the Texas Engineering Experiment Station has been engaged in a two year program to define and study problems associated with the design of free balloons. This work was prompted by the observation that no significant changes have been made in the design procedure of free balloons in the last decade while at the same time increases in payload and altitude capabilities were such that the growth potential of these systems appeared to be limited. Therefore, a strong engineering effort was mounted to identify the technological voids and to attempt to fill those areas considered necessary to advance the state-of-the-art.

The nature of this program was such that both analytic and experimental studies were necessary. The interdisciplinary approach involved three members of the professional staff supported by a number of students, clerical and technical personnel. A list of those who contributed to this program is contained in Appendix A of this report.

The problems identified by this study were twofold. The shape and stress distributions in a typical balloon disign did not include the effects of load tape stiffness, material deformation or lobing. The results of this study have been adequately documented in Scientific Report No. 1. The characterization of balloon films was found to be lacking and required an extensive experimental study of frequently used balloon materials. The results of this study have been documented in Scientific Report No. 2.

A number of national presentations and publications have been generated under the sponsorship of this program. A complete list of these is contained in Appendix B of this report. The purpose of this report is to summarize the significant results of this program and document in detail only those items which have not been previously published.

Diaphragm Analysis

In order to properly evaluate biaxial test data obtained from circular diaphragm testing devices, an analysis of the test was considered appropriate. Since deformations are quite large relative to the film thickness, large deformation, non-linear theory was required. A brief review of the literature indicated that a modified Ritz procedure would be adequate for the five inch radius circular, axisymmetric membrane testing device used by the Mechanics and Materials Laboratory. The procedure, adequately described in Appendix C, assumes that the material is incompressible and can be characterized by a Mooney strain-energy density function, and that the shape is described by assumed position functions instead of the usual, small deformation, displacement functions.

Results were obtained for one mil Stratofilm® which was modeled as a neo-Hookean material with the correct uniaxial modulus at room temperature. The stresses, strains and shape of the diaphragm were computed for pressure differentials from .05 to .5 psi with the following results:

a. The shape of the deformed surface is "nominally" spherical. Visual inspection of the shape will not reveal the inherent departure from spherical shape and only precision measurements will detect this departure.

b. The strains predicted by assuming a spherical shape are relatively good estimates of the meridional strain. However, the circumferential strain is zero at the edge restraint and equal to the meridional strain only at the center.

c. The meridional stresses predicted by assuming a spherical shape are lower than those computed by 10 to 15 percent depending on the pressure. In addition, the meridional stress is not constant, as is normally assumed, but varies by more than 25 percent along the radius.

d. The circumferential stress is equal to the meridional stress only in a small region near the center of the diaphragm.

This stress then decreases by more than 60 percent at the edge restraint.

e. Results have been obtained for relatively small deformations and the error is seen to increase with bubble height. It is concluded that the errors inherent in assuming a spherical shape will become even worse as the material yields.

f. Failure of the diaphragm away from the center indicates an inplane failure mechanism due to the difference in principal stresses. This would be characterized as an "in-plane" shear failure even though the "through-plane" shear stress is higher at the center.

In addition, a correction factor has been developed for use by laboratory personnel which will allow the estimation of the biaxial stress present at the center of the diaphragm. However, this correction is based on the assumption that the membrane material is isotropic in the plane of the diaphragm and should not be used if there are significant differences in the uniaxial modulii in the machine and transverse directions.

Design Procedure

The design of free balloons for scientific application has received the attention of many investigators over the years. The well documented shape and stress calculations of Smalley (1,2) represented the state-of-the-art and was the starting point for the present study. The purpose of this study was to analyze the assumptions that are necessarily made in the design process in order to assess the impact of these assumptions on the final design product.

In order to obtain a more realistic estimate of the state of stress, the design equations were rederived in a manner to take into account the effects of load tape stiffness and film modulus. This was accomplished by making the usual assumptions regarding symmetry and prescribing a desired circumferential stress. In addition, it was assumed that both tape and film were linearly elastic and the meridional strain in both tape and film were equal. The details of this derivation are included in Scientific Report No. 1 (Appendix B).

The significant result of this procedure is a more realistic estimate of the meridional stress distribution due to the presence of the load tapes. This stress is considerably less than that predicted from the tapeless assumption and always remains finite. The difference in the meridional forces between the two procedures is a measure of the force carried by the load tapes.

This design procedure was used to identify the effects of the number of load tapes and film thickness on the final balloon size by Keese in his paper, "Zero Pressure Balloon Design" (Appendix B). Although his conclusions may have been premature, his approach appears to be the first documented attempt to utilize stress analysis as a decision parameter in the shape design process.

Balloon Stress Analysis

The stress analysis of a flexible structure is significantly different from the design problem. Once the balloon has been manufactured, the available material at each gore position is fixed and the solution should be obtained as a boundary value problem rather than as an initial value problem. In addition, the simplifying assumptions of zero circumferential stress and a surface of revolution should not be made since it is suspected that this will significantly alter the state of stress.

In order to obtain a realistic state of stress a load transfer mechanism has been proposed which will permit the transfer of forces between the tapes and the film. The presence of a shear stress along the edge of the tape is essential if the force in the tape is to be permitted to change. In addition, if the tape is to assume the same shape as the film, the lobing in conjunction with a circumferential stress may be considered to be a proper mechanism to produce this type of compatibility of shape. It is interesting to note that if the design shape is assumed to be correct, neither the tape nor the film will satisfy the equilibrium equations independent of circumferential stress.

A new formulation of the analysis problem has been developed which incorporates a number of features that should result in a realistic state of stress. Lagrangian coordinates are routinely used in addition to "engineering" stress and strain. This effectively results in the simplification that film thickness and deformed gore positions need not be considered during the solution process. The equilibrium equations have been derived assuming the film to lobe in a plane containing meridional radius of curvature to two adjacent load tapes. In addition, the load transfer mechanisms described in the preceeding paragraph have been included in the model.

A variety of solution schemes were attempted on the resulting boundary value problem since numerical instabilities were quite prevalent. However, a solution technique known as "direct iteration" proved to be successful when used with a "relaxation" technique. In this technique, a starting solution is required for all variables and then the corrections to the assumed solution are obtained from the model. Only a portion of the full correction is used to update the solution and the procedure is then repeated until the corrections reach an acceptably low value.

This new formulation was applied to a typical heavy load balloon design which is of interest to a variety of organizations. The results of this analysis are described in detail in Scientific Report No. 1 (Appendix B). However, the results may be summarized as follows:

a. Both meridional and circumferential stress distributions in the capped regions of the balloon considered are significantly different than those reported by Alexander (3).

b. The maximum circumferential stress in this balloon occurs at the edge of the cap and is of the same order of magnitude as the meridional stress.

c. The stress levels increase as the balloon is pulled to lower altitudes by increasing the payload.

The equilibrium model has been programmed in such a way as to permit the load tapes to unload completely. This will occur if sufficient circumferential stress is developed to create a Poisson effect in the meridional direction. Under these conditions, the total meridional load must be carried by the film and the tapes are assumed to be slack. This condition has apparently been observed by several persons experienced in balloon operations.

An essential element in predicting details of this nature is the accurate description of the stress-strain behavior operative at the time. In this analysis the material was assumed to be linearly elastic and orthotropic characterized by the equations:

$$\varepsilon_m = D_m \sigma_m + D_{mc} \sigma_c \qquad [1]$$

$$\varepsilon_c = D_{mc} \sigma_m + D_c \sigma_c \qquad [2]$$

The results to date were obtained by assuming that the material was incompressible isotropic, and the material constants were uniquely determined from the temperature. It should be expected that significant departures from the sample results will occur if the material properties are assumed to be functions of strain rate and temperature histories.

Film Characterization

In order to properly charcterize thin films of interest, a series of well controlled uniaxial tests were run at a variety of constant temperatures and strain rates. Winzen Research Inc. provided lots of their material, Stratofilm R, in thicknesses of .5, .7, and 1 mil. These films were then tested in two directions at four rates of strain from .2 to 200 inches/inch/minute. Each of these were tested at temperatures of -80°C, -23°C, +23°C, and +38°C. Five specimans, one inch by ten inches in length were pulled to obtain load-deformation characteristics in the elastic and early plastic region for each rate and temperature. The resulting data were reduced to stress-strain data characteristic of each film thickness and direction.

Another useful test which may be helpful in the characterization of viscoelastic materials is the creep - creep recovery test. In this procedure the speciman is held under a state of uniaxial stress and the deformation measured as a function of time. Tests of this type may take weeks to perform but yield compliance data suitable for mathematical modeling. At some prescribed time, the stress is removed and the deformation again measured as a function of time. This recovery data is also useful in developing the mathematical model. Creep - creep recovery tests were performed on Statofilm at stress levels of 500 and 1000 psi and at temperatures of -23°C, +23°C, and 38°C. These were the temperatures of the environmental rooms available for testing.

An attempt was then made to develop a mathematical model which would allow the uniaxial performance of this film to be predicted at a variety of rates and temperatures. It was quickly determined that there was no statistical difference between machine direction data and transverse direction data. In addition it was difficult to discern any difference in the various thicknesses of film used in the testing program. Finally, it was determined that the normally acceptable viscoelastic shift procedures did not apply to this material. It is postulated that significant crystallinity exists in this material and thus precludes the use of standard techniques.

Therefore, an empirical shift factor was developed to give a reduced relaxation modulus curve that would at least fit the results of both creep tests and constant rate tests. This relationship may be expressed in the following form if the reference temperature of 0°C is selected:

$$\log a_T = -0.116T + 2.668 \qquad [3]$$

where a_T is in minutes. The modulus at any time or temperature may then be expressed in the form:

$$E(t) = E_1 \; (t/a_T)^n \qquad [4]$$

The Stratofilm data was found to be represented by E_1 = 20,000 psi and n = -0.1. The results of this study have been documented in Scientific Report No. 2 (Appendix B). One conclusion obtained in this study is the possibility that constant rate data may be a more sensitive measure of the material properties and permit a better representation than creep data would yield. This is due to the fact that large deviations in strain will cause only modest changes in properties obtained from logarithmic plots of this data. At the same time, use of small strain, high rate data to predict long term creep behavior may be somewhat presumptuous. An examination of the limits of the constant rate data presented in Scientific Report No. 2 indicates that it cannot be represented by the simple power law as proposed.

Laboratory Testing

The experimental investigation of thin films is a unique challenge to laboratory ingenuity. Although some commonly accepted test techniques such as the uniaxial, constant rate, test may provide valuable information, other techniques have had to be devised to produce results with acceptable accuracy. The measurement of film thickness may be a formidable problem if sufficient care in not taken in the area of cleanliness and calibration of the instruments used. In addition, the variability obtained in multiple tests has dictated the need for many tests to be performed to obtain even average data. It is common practice to perform a minimum of five identical tests before a repeatable results is obtained. As a result a number of new testing techniques have been developed. These will be briefly described here and are thoroughly described in Scientific Report No. 2.

Due to the large number of tests required and the time necessary to reduce the raw data into a form useful for engineering purposes, the need for a computerized data acquisition and data reduction capability was recognized. The concurrent availability and economy of microprocessors provided the solution. The laboratory was equipped with a microprocessor, with sufficient memory and storage capability to obtain reduced, average, stress-strain-time figures from any of the testing devices including the uniaxial testing machine, the 10 inch diaphragm tester and the racetrack testing device.

The racetrack testing procedure was developed many years ago to produce a biaxial state of stress and material failure independent of gripping conditions. However, due to the large number of tests needed to guarantee repeatability, especially at low temperatures, the old testing procedure was considered too slow. In order to clamp the speciman, reduce the chamber temperature to a uniform value, prevent ice formation, and pressurize the system to failure, one hour was required. A new facility was designed and fabricated which permits the race-track speciman to be cooled, clamped, tested and data reduced and plotted in less than five minutes. In essence, the system involves the insertion of the speciman in a low thermal mass grip into a precooled chamber. The grip is then pneumatically clamped into place after the speciman cools. The microprocessor is then used to control the pressurization-rate, acquire and store the raw data. After sufficient tests have been performed, the computer then reduces the data, eliminates bad data points, and produces the average stress-strain characteristics of the material.

In order to test film and tape samples with a predetermined strain field, a special test apparatus was designed and fabricated. The cruciform tester is a device which may be attached to a standard Instron machine. The scissors like motion of the links which hold the speciman allows the material to be subjected to a known biaxial strain rate. Heat seals have been successfully tested with this device which has demonstrated the strength of such a tape-film joint.

A variety of special tests were performed demonstrating the practicality of some of the results of the materials research program. In particular:

1. A proposed material for use in a tethered balloon, DV808D-11, was tested to determine its uniaxial stress-strain behavior, tear strength, and gas permeability.
2. The question of path dependency was demonstrated to be a proper concern for polyethylene by means of two tests in which the temperature-stress histories were revised.
3. Film from a polyethylene balloon (stored for twenty two years) was tested to determine if there were detrimental aging effects. The data showed no such effects and the balloon appeared suitable for flight.

Further Investigations

Much progress has been made in the areas of Balloon stress analysis, materials testing and characterization. However, a variety of areas have now been identified as potentially critical and a beginning has been made to develop the tools necessary to attack these problem areas.

The ability to analyze the state of stress in the off-design configuration should be expanded to consider the entire flight environment. Since path dependency has been demonstrated to influence the behavior of polyethylene films, it would seem obvious that this would be the primary area of concern in the future.

The state of stress from launch, during ascent through the tropopause, to the final float configuration should identify the critical design conditions. Due to the relatively poor performance of "heavy" load balloons, it would appear that a concentrated effort in this area is warranted. At the present time, the analysis program assumes the film may be characterized by a linearly, elastic orthotropic material. Although this is a biaxial characterization, it is not time dependent. Therefore, a time-temperature-biaxial stress characterization of thin films would be an essential element in any further successful analysis attempt.

REFERENCES

1. Smalley, Justin H.; Determination of the Shape of a Free Balloon; Litton Systems, Inc. Rept. No. 2713, AFCRL-65-92, (1965).

2. Smalley, Justin H.; Balloon Shapes and Stresses Below the Design Altitude; NCAR-TN-25, (1966).

3. Alexander, H. and Agrawal, P.; Gore Panel Stress Analysis of High Altitude Balloons; Stevens Institute of Technology, AFCRL-TR-74-0597, (1974).

APPENDIX A

Personnel

I. Professional

A. E. Cronk - Principal Investigator

James L. Rand - Principal Investigator and Research Engineer

L. Dale Webb - Research Associate

II. Staff

Russell S. Adams	-	Student Worker IV
Robert H. Bishop	-	Student Worker III
Frank A. Boyle	-	Student Worker I
Susan G. Caldwell	-	Senior Secretary
James F. Echols	-	Student Worker IV
Carl E. Fredericksen	-	Electronics Tech II
Robert R. Hafernik Jr.	-	Student Worker III
Tom H. Hooper	-	Student Worker III
David L. Keese	-	Student Worker IV
Michael W. Lehman	-	Student Technician
David W. Lund	-	Student Worker III
F. Scott Macaluso	-	Coop Research Aide
Timothy W. Morse	-	Student Worker IV
Kimberly A. Osborn	-	Student Worker III
Marla G. Painter	-	Student Worker III
Joe L. Reed	-	Student Technician
Larry E. Perry	-	Student Technician
Larry A. Schulze	-	Student Worker II
George P. Sladecek	-	Student Technician
Janis L. Sloan	-	Secretary
Curtis D. Symank	-	Student Technician
Edward Y. Teng	-	Student Worker III

APPENDIX B

Publications and Presentations

Publications

1. Zero Pressure Balloon Design; Keese, D. L., AIAA Paper No. 78-314, AIAA 14th Annual Meeting and Technical Display, Washington, D.C., Feb. 1978.

2. Analysis of Balloons in Off-Design Configurations; Rand, J.L.; Proceedings of the Tenth AFGL Scientific Balloon Symposium; Portsmouth, N.H., Aug. 1978.

3. Design and Analysis of Single Cell Balloons; Rand, J.L.; Scientific Report No. 1, AFGL-TR-78-0258, TAMRF-3332-1, Aug. 1978.

4. Mechanical Behavior of Balloon Film; Webb, L.D.; Scientific Report No. 2, AFGL-TR-79-0026, TAMRF-3332-2, Nov. 1978.

5. Define and Study Free Balloon Design Problems; Rand, J.L.; Final Report, AFGL-TR-78-0295, TAMRF-3332-3, Nov. 1978.

6. Zero Pressure Balloon Design; Keese, D.L., Vol. 17, No. 1 AIAA Journal, Jan. 1979.

Presentations

1. Zero Pressure Balloon Design by D. L. Keese
   a. 25th Annual AIAA Southwest Student Paper Competition; Ft. Worth, TX, April 1977.
   b. AIAA National Student Conference, held in conjunction with AIAA 14th Annual Meeting and Technical Display, Washington, D. C., Feb. 1978.

2. Analysis of Balloons in Off-Design Configurations by J. L. Rand. Presented at the 10th AFGL Scientific Balloon Symposium, Wentworth-by-the Sea, Aug. 1978.

APPENDIX C

Diaphragm Analysis

The purpose of this study was to evaluate the capability of this testing procedure to produce meaningful data which could subsequently be used in the characterization of polyethylene balloon films. It is hoped that this study will contribute to an improved understanding of that state of stress in balloon films and assist in the development of a biaxial failure criteria which is in agreement with both laboratory results and flight experience.

Circular diaphragm testers have been used for many years to create a biaxial state of stress in balloon films. When a pressure differential is applied to the film the initially flat diaphragm forms a bubble which appears to be spherical. It has been common practice to assume this deformed shape to have a radius of curvature which may be calculated by simply measuring the bubble height, h_o, and assuming a spherical shape. The radius of curvature is then given by the relation:

$$R = \frac{h_o}{2} + \frac{x_o^2}{2h_o}$$

where x_o is the radius of the circular clamp.
The biaxial stresses in the film are then assumed equal and given by the relation:

$$\sigma t = \frac{pR}{2}$$

The meridional extension ratio (or strain) may also be computed from the radius of curvature as:

$$\lambda = \frac{R}{x_o} \sin^{-1} \left(\frac{x_o}{R} \right)$$

This test has been used in several laboratories with a variety of results. Hauser has used this test to obtain biaxial material properties. He noted that in several cases the bubble was not spherical and developed several techniques to measure the radius of curvature near the center. Electrical devices and templates were used but this only led to suspicion of the test itself. Alexander noted that the clamp causes the circumferential strain at the boundary to be zero which prevents the biaxial stresses from being balanced. He then developed the racetrack test which supposedly causes a known stress state at least in the center of the fixture. Webb has used the circular diaphragm and racetrack testers to "completely characterize" several balloon films. In addition the tester is used for comparative studies of different films. The results to date have been inconsistent with uniaxial data.

The deformation of a circular membrane under constant pressure is a classic problem which has attracted international interest for many years. The problem is quite challenging since the deformations are large (compared to the thickness) which makes the governing equations non-linear and the material properties in their simplest form are non-linearly elastic (not to mention the visco-plastic nature of the material). The following is a partial list of the material that is available in our library on this topic:

1) Flint, C.F. and Naunton, W.J.S.; Institution of Rubber Industries, Transactions; Vol. 12, p. 59, (1937).
2) Treloar, L.R.G.; Transactions of the Faraday Society; Vol. 40, p. 59, (1944).
3) Treloar, L.R.G.; Institution of Rubber Industries, Transactions; Vol. 19, p. 201, (1944).
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5) Levinson, M.; Journal of Applied Mechanics, Vol. 32; Transactions of the ASME, Vol. 87, Series E, p. 656, (1965).
6) Hart-Smith, L.J. and Crisp, J.D.L.; International Journal of Engineering Science, Vol. 5, P. 1, (1967).
7) Yang, W.H. and Feng, W.W.; Journal of Applied Mechanics, Vol. 37, Transactions of the ASME, Vol. 92, Series E, p. 1002, (1970).
8) Glockner, P.G. and Vishwanath, T.; International Journal of Non-Linear Mechanics; Vol. 7, p. 361, (1972).
9) Tielking, J. T. and Feng, W.W.; Journal of Applied Mechanics, Vol. 96, Series E, p. 491, (1974).

Without going into detail, the Ritz energy minimization technique was selected as the most straight forward and powerful in that it contains the ability to solve different geometric boundary conditions by changing the assumed position functions. The details of this technique are contained in an excellent article by Tielking and only the essential details of that formulation will be presented here. The film is assumed to be incompressible in the thickness direction, as well as homogeneous and isotropic. The stress-strain behavior of the material is assumed to be given by the Mooney-Rivlin energy density function, U, given by the relation:

$$U = C_1\left[(I_1-3) + \alpha(I_2-3)\right]$$

The functions, I_1 and I_2, are the first and second strain invariants expressed in terms of the extension ratios, λ_1 and λ_2, as:

$$I_1 = \lambda_1{}^2 + \lambda_2{}^2 + \frac{1}{\lambda_1{}^2\lambda_2{}^2}$$

$$I_2 = \lambda_1{}^2\lambda_2{}^2 + \frac{1}{\lambda_1{}^2} + \frac{1}{\lambda_2{}^2}$$

The stress resultants in the two directions, T_1 and T_2, are given by:

$$T_1 = h\frac{\partial U}{\partial \lambda_1} \qquad \text{and} \qquad T_2 = h\frac{\partial U}{\partial \lambda_2}$$

where h is the initial film thickness. The method consists of expressing the final position of the film in terms of a Fourier series with unknown coefficients. If the series were taken to the limit, an exact solution would result but truncation can be accomplished without introducing a significant error. In this case the final position is expressed as:

$$x(r) = r + \sum_{n=1}^{8} A_n \sin n\pi r$$

$$y(r) = \sum_{n=1}^{8} B_n \cos(n-\tfrac{1}{2})\pi r$$

Here r is the original position of a point on the diaphragm while x and y are the final radial and height positions of the point. The total potential, Π, is defined in terms of the potential due to internal forces (or strain energy) and the potential due to external forces (or the negative of the work done by the pressure in causing the deformation). This may be expressed as:

$$\Pi \equiv \int_{Vol} U\, dv - W$$

where

$$W \equiv \pi p \int x^2\, dy = \pi p \int_0^R x^2 y^1\, dr$$

Since the extension ratios may be expressed in terms of x, y and their derivatives then the total potential may be expressed entirely in terms of the unknown coefficients, A_n and B_n. The correct final position is obtained by minimizing the total potential with respect to the unknown coefficient. Therefore, 16 independent equations are formed by setting the variation of Π equal to zero:

$$\frac{\partial \Pi}{\partial A_n} = 0 \qquad n=1, 2 \ldots, 8$$

$$\frac{\partial \Pi}{\partial B_n} = 0 \qquad n=1, 2 \ldots, 8$$

These equations are then solved simultaneously for the 16 unknown coefficients. This then defines the final position of each point on the diaphragm as well as the stresses and strains at each point.

In order to characterize polyethylene film in terms of the Mooney-Rivlin energy dens'ty function it is necessary to understand the influence of the material constants C_1 and α on the stress strain behavior of the material. By taking the necessary derivatives it may be shown that the two stresses are given by:

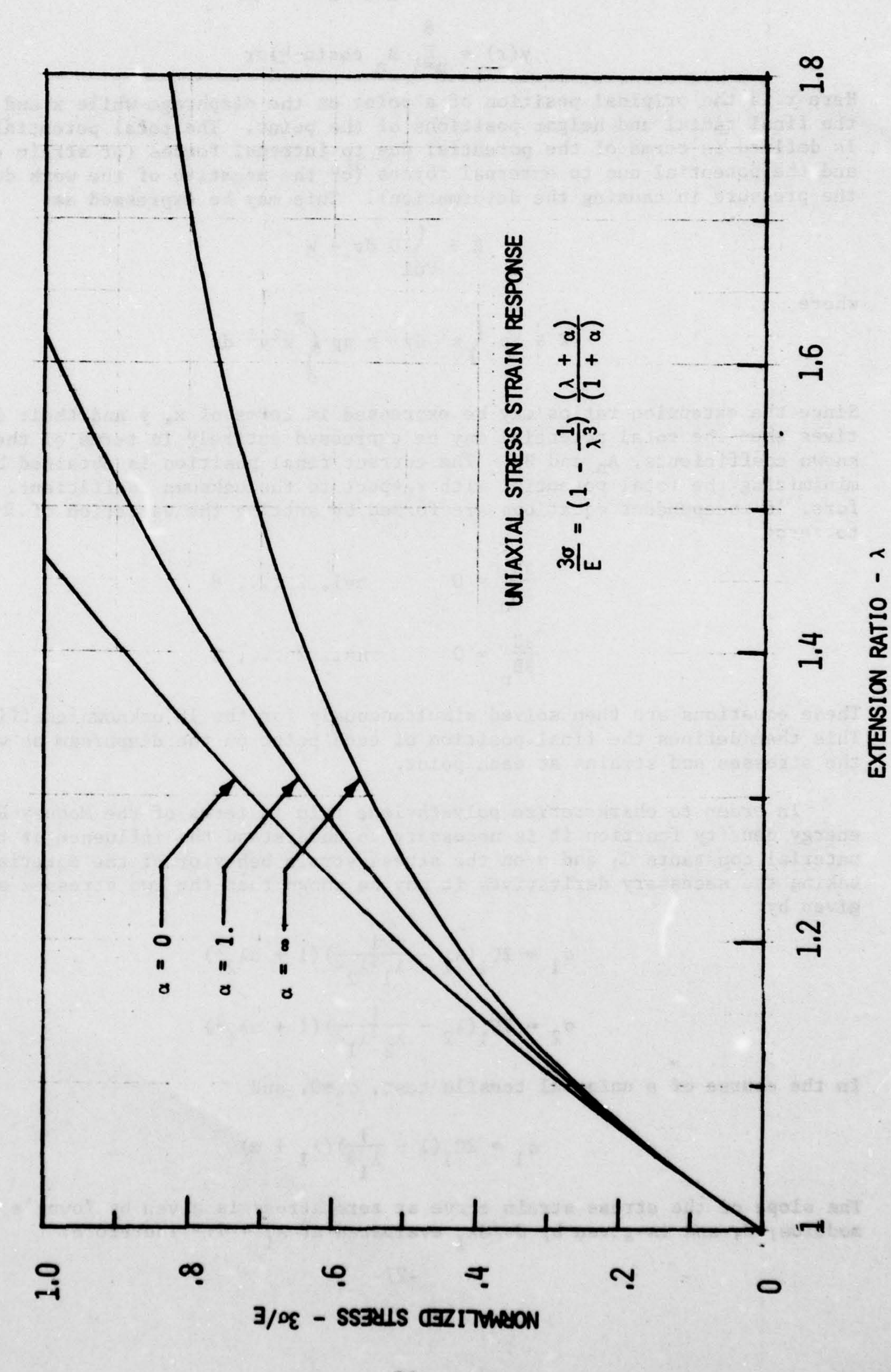
$$\sigma_1 = 2C_1(\lambda_1 - \frac{1}{\lambda_1^3\lambda_2^2})(1 + \alpha\lambda_2^2)$$

$$\sigma_2 = 2C_1(\lambda_2 - \frac{1}{\lambda_2^3\lambda_1^2})(1 + \alpha\lambda_1^2)$$

In the course of a uniaxial tensile test, $\sigma_2=0$, and

$$\sigma_1 = 2C_1(1 - \frac{1}{\lambda_1^3})(\lambda_1 + \alpha)$$

The slope of the stress strain curve at zero stress is given by Young's modulus, E, and is given by $d\sigma/d\lambda_1$ evaluated at $\lambda_1 = 1$. Therefore:


UNIAXIAL STRESS STRAIN RESPONSE
$\frac{3\sigma}{E} = (1 - \frac{1}{\lambda^3}) \frac{(\lambda + \alpha)}{(1 + \alpha)}$
α = 0
α = 1.
α = ∞
NORMALIZED STRESS - 3σ/E
EXTENSION RATIO - λ
0
.2
.4
.6
.8
1.0
1
1.2
1.4
1.6
1.8

$$\left.\frac{d\sigma}{d\lambda_1}\right|_{\lambda_1=1} = 6C_1(1+\alpha) = E$$

In order for the stress strain response to have this essential property then:

$$\sigma_1 = \frac{E}{3}\left(1 - \frac{1}{\lambda_1^3}\right)\frac{(\lambda_1 + \alpha)}{(1 + \alpha)}$$

This equation is shown in the attached figure. Although the response is highly non-linear for large strains, the region near the origin is reasonably linear and independent of the material constant α. Therefore, when only small strains are involved it is assumed that $\alpha=0$ and $C_1=E/6$. This is called a Neo-Hookean representation and should be reasonably accurate.

Young's modulus for polyethylene balloon film has been reported by Alexander as a function of temperature. Values were selected at a variety of temperatures of interest and are presented in the accompanying table.

T(°C)	40	20	0	-20	-40	-60
E(psi)	10,000	18,000	30,000	74,000	165,000	267,000
C_1(psi)	1,667	3,000	5,000	12,333	27,500	44,500

The solution technique that has been briefly described above has been applied to a variety of problems to investigate the effects of diaphragm radius, pressure, thickness and material properties. In addition to yielding information on the response of a diaphragm, several interesting features of the technique itself were observed.

Effects of Diaphragm Radius - The Texas A&M University Mechanics and Materials Laboratory utilizes a five inch radius diaphragm tester. Therefore, all results reported here will be based on a five inch diaphragm with this one exception. Due to the non-linear nature of the problem the response of a unit radius diaphragm was computed for comparison purposes. The material was assumed to be one mil Stratofilm at room temperature, i.e. C_1=3000 psi; and t=.001 inch. The pressure was assumed to be 0.1 psi in both cases. A sample of the computer output for the five inch radius is given in the accompanying table. The computed shapes of these two diaphragms are presented in the attached figure. In both cases the shape has been normalized with respect to the diaphragm radius. It should be noted that the shape and therefore the stresses and strains cannot be scaled directly from the results for a unit radius diaphragm under the same conditions but must include some function of pressure.

The stress distributions in the five inch diaphragm is presented in the next figure. These distributions are typical of that to be expected in this type of test. It should be noted that the meridional stress is equal to the circumferential stress only at the center of the film. This stress is significantly greater than that predicted by assuming a spherical shape. The effect of the restrained boundary is apparent due to the decrease in stress as the boundary is approached. In addition the difference in the meridional and circumferential stresses indicated the pressence of a significant shear stress

CONVERGENCE WITH ITERATION 5

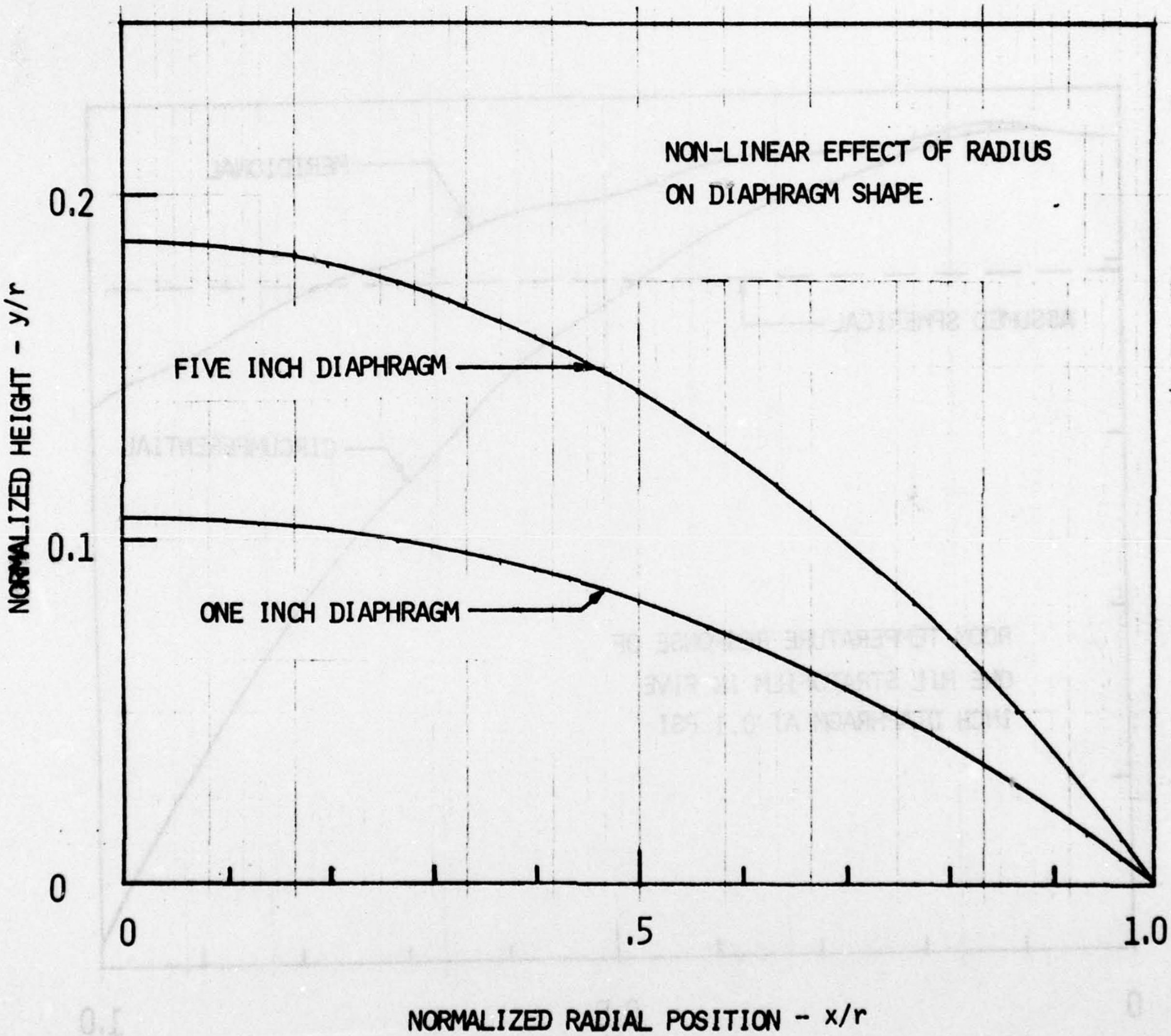
0.47376E-01 -0.79462E-02 0.25833E-02 -0.11377E-02 0.58871E-03 -0.34700E-03 0.21116E-03 -0.11996E-03

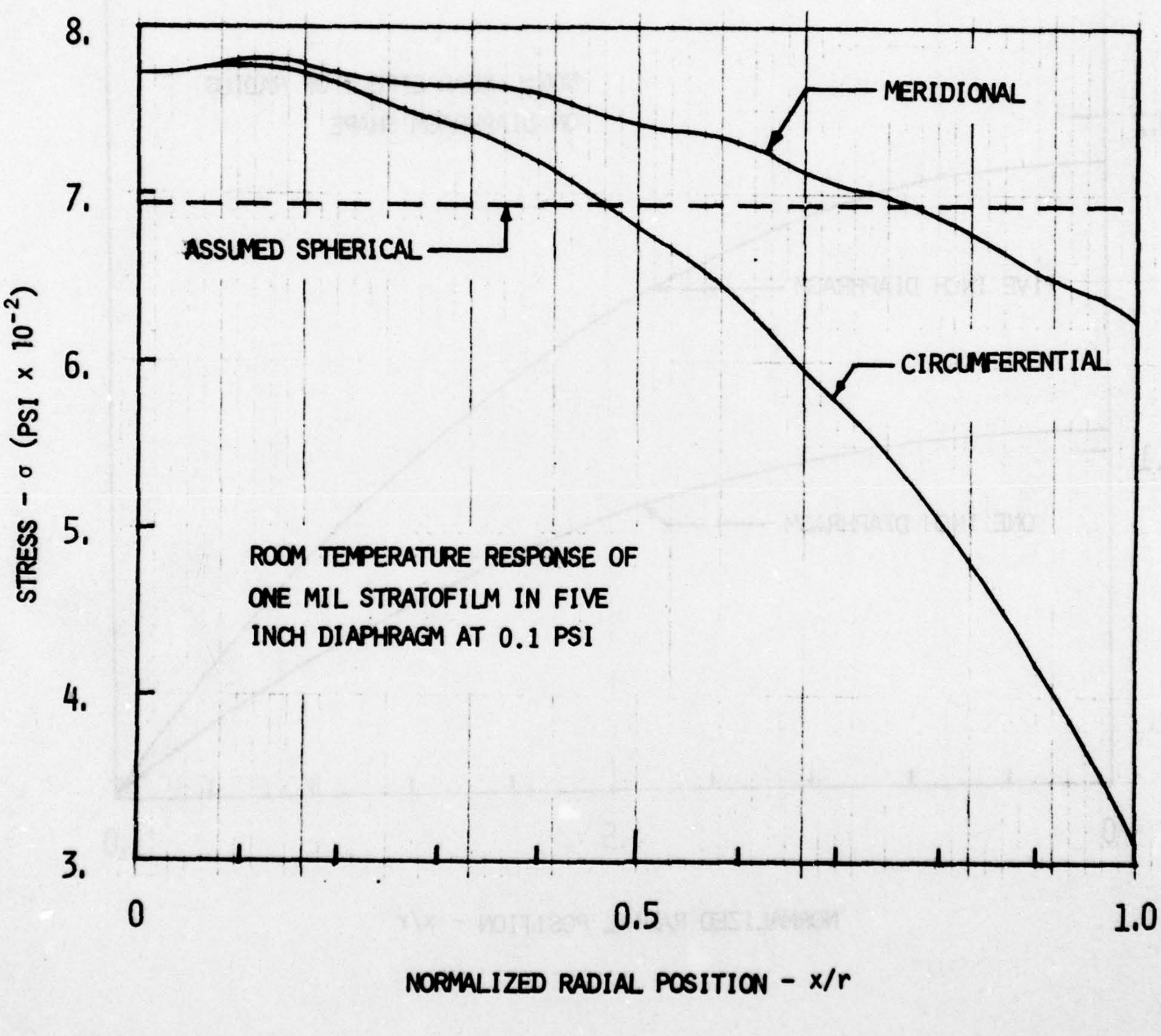
0.97412E 00 -0.49776E-01 0.11431E-01 -0.43803E-02 0.19066E-02 -0.12275E-02 0.58136E-03 -0.61561E-03

FOR P = 0.100 ALPHA = 0.000 CIH = 3.000 R = 5.000

N	S	X	Y	SR1	SR2	T1	T2
1	0.0000	0.0000	0.9321	1.0227	1.0227	0.7717	0.7717
2	0.1667	0.1704	0.9313	1.0227	1.0227	0.7734	0.7727
3	0.3333	0.3409	0.9289	1.0229	1.0227	0.7772	0.7748
4	0.5000	0.5113	0.9245	1.0231	1.0227	0.7806	0.7763
5	0.6667	0.6817	0.9181	1.0231	1.0226	0.7812	0.7753
6	0.8333	0.8520	0.9095	1.0231	1.0224	0.7787	0.7714
7	1.0000	1.0222	0.8987	1.0230	1.0222	0.7741	0.7651
8	1.1667	1.1922	0.8860	1.0229	1.0219	0.7696	0.7580
9	1.3333	1.3621	0.8714	1.0230	1.0216	0.7673	0.7513
10	1.5000	1.5316	0.8551	1.0231	1.0212	0.7665	0.7449
11	1.6667	1.7014	0.8369	1.0233	1.0208	0.7658	0.7352
12	1.8333	1.8707	0.8168	1.0234	1.0204	0.7633	0.7299
13	2.0000	2.0398	0.7944	1.0234	1.0199	0.7586	0.7194
14	2.1667	2.2086	0.7698	1.0234	1.0193	0.7525	0.7071
15	2.3333	2.3770	0.7430	1.0234	1.0187	0.7468	0.6939
16	2.5000	2.5452	0.7142	1.0235	1.0181	0.7425	0.6807
17	2.6667	2.7130	0.6835	1.0237	1.0174	0.7393	0.6673
18	2.8333	2.8804	0.6508	1.0239	1.0166	0.7357	0.6531
19	3.0000	3.0475	0.6159	1.0241	1.0158	0.7301	0.6370
20	3.1667	3.2141	0.5786	1.0241	1.0150	0.7224	0.6185
21	3.3333	3.3800	0.5387	1.0242	1.0140	0.7139	0.5981
22	3.5000	3.5454	0.4963	1.0244	1.0130	0.7064	0.5767
23	3.6667	3.7102	0.4516	1.0246	1.0119	0.7003	0.5550
24	3.8333	3.8744	0.4046	1.0249	1.0107	0.6947	0.5325
25	4.0000	4.0380	0.3554	1.0252	1.0095	0.6877	0.5083
26	4.1667	4.2009	0.3037	1.0254	1.0082	0.6778	0.4812
27	4.3333	4.3628	0.2491	1.0256	1.0068	0.6662	0.4512
28	4.5000	4.5236	0.1921	1.0259	1.0053	0.6558	0.4192
29	4.6667	4.6833	0.1298	1.0264	1.0036	0.6479	0.3859
30	4.8333	4.8419	0.0657	1.0269	1.0018	0.6394	0.3506
31	5.0000	5.0000	0.0000	1.0271	1.0000	0.6246	0.3122

SAMPLE OUTPUT


NON-LINEAR EFFECT OF RADIUS
ON DIAPHRAGM SHAPE
FIVE INCH DIAPHRAGM
ONE INCH DIAPHRAGM
NORMALIZED HEIGHT - y/r
0.2
0.1
0
0
.5
1.0
NORMALIZED RADIAL POSITION - x/r


8.
7.
6.
5.
4.
3.
STRESS - σ (PSI x 10^-2)
MERIDIONAL
ASSUMED SPHERICAL
CIRCUMFERENTIAL
ROOM TEMPERATURE RESPONSE OF
ONE MIL STRATOFILM IN FIVE
INCH DIAPHRAGM AT 0.1 PSI
0
0.5
1.0
NORMALIZED RADIAL POSITION - x/r

near the boundary. Physically these results may be interpreted to mean that the bubble, although "appearing" spherical, has a reduced curvature near the boundary and an increased curvature near the center of the diaphragm.

The small oscillation which appears in the stress distributions is somewhat disconcerting and is a result of the numerical method. The truncation of each of the position function series at eight terms can be shown to cause this feature of the solution. It gives the appearance that the maximum stress occurs slightly off center which is not the case. It is caused by the fact that the sine function is an odd function which must always increase in value from the origin. If the oscillation is averaged out, the maximum stress will occur at the origin and is approximately equal to the stress value at N=5. The problem was solved with a six term series which gave essentially the same results; however, the magnitude of the oscillation was increased and the number of cycles was reduced from four to three. Therefore, all additional cases were solved utilizing the eight term position functions previously mentioned.

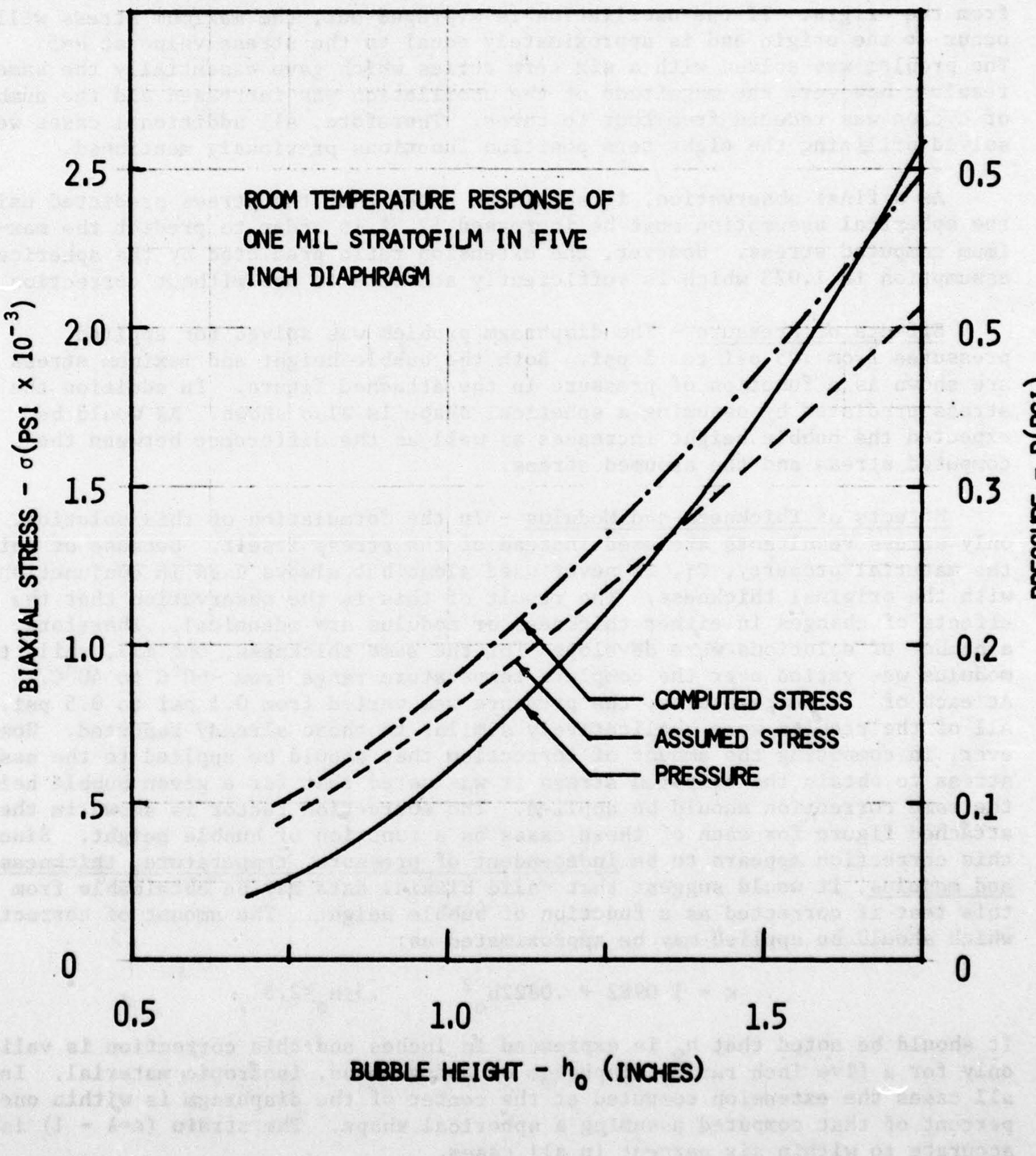
As a final observation, it should be noted that the stress predicted using the spherical assumption must be increased 12.5% in order to predict the maximum computed stress. However, the extension ratio predicted by the spherical assumption is 1.023 which is sufficiently accurate to use without correction.

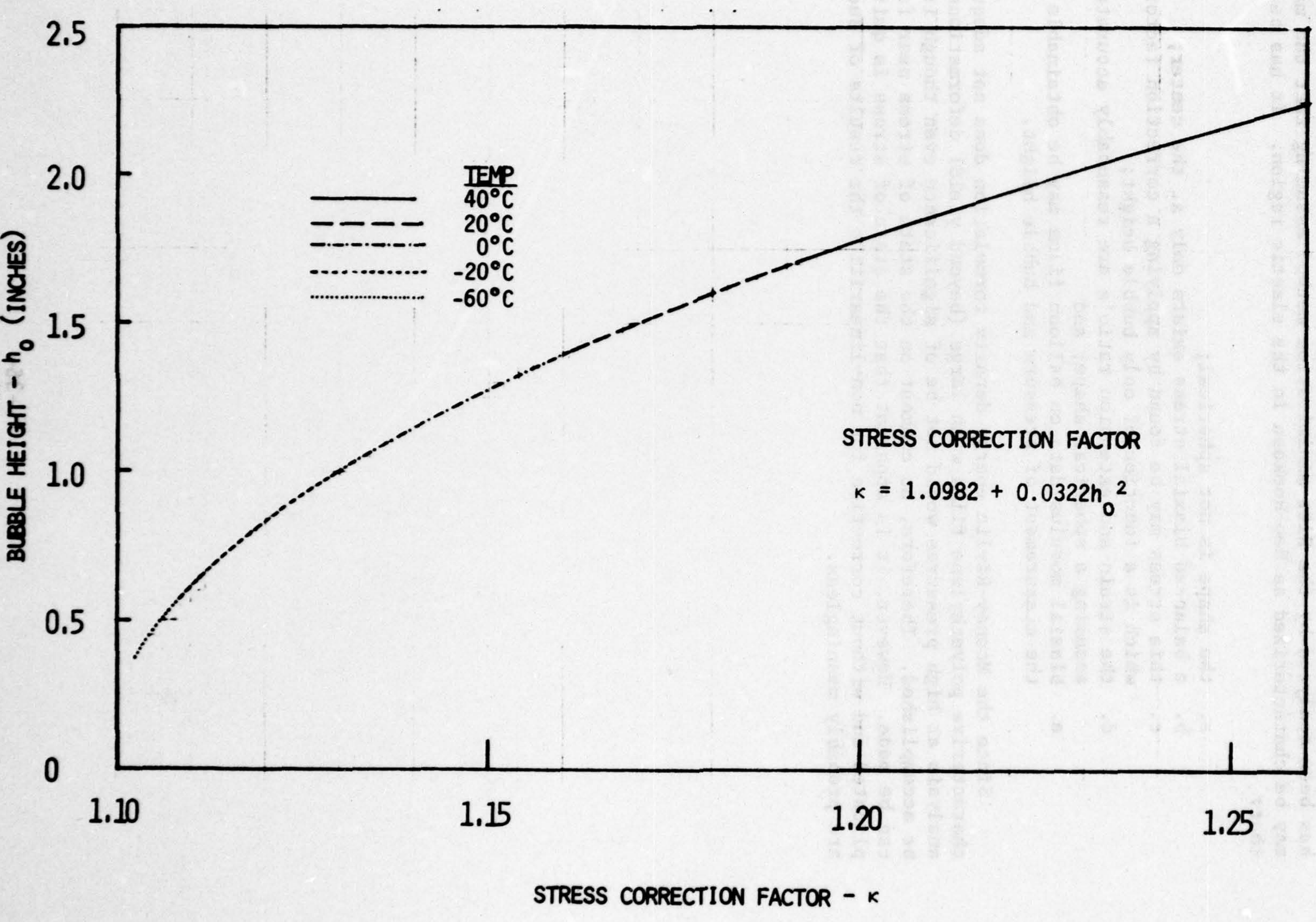
Effects of Pressure - The diaphragm problem was solved for applied pressures from .05 psi to .5 psi. Both the bubble height and maximum stress are shown as a function of pressure in the attached figure. In addition the stress predicted by assuming a spherical shape is also shown. As would be expected the bubble height increases as well as the difference between the computed stress and the assumed stress.

Effects of Thickness and Modulus - In the formulation of this solution, only stress resultants are used instead of the stress itself. Because of this, the material property, C_1, is never used alone but always used in conjunction with the original thickness. The result of this is the observation that the effects of changes in either thickness or modulus are identical. Therefore, a number of solutions were developed for the same thickness, one mil, while the modulus was varied over the complete temperature range from -60°C to 40°C. At each of the temperatures, the pressure was varied from 0.1 psi to 0.5 psi. All of the results were qualitatively similar to those already reported. However, in computing the amount of correction that should be applied to the assumed stress to obtain the computed stress it was noted that for a given bubble height, the same correction should be applied. The correction factor is shown in the attached figure for each of these cases as a function of bubble height. Since this correction appears to be independent of pressure, temperature, thickness and modulus, it would suggest that valid biaxial data may be obtainable from this test if corrected as a function of bubble height. The amount of correction which should be applied may be approximated as:

$$\kappa = 1.0982 + .0322 h_o^2 \qquad .5 \le h_o \le 2.5$$

It should be noted that h_o is expressed in inches and this correction is valid only for a five inch radius diaphragm of homogeneous, isotropic material. In all cases the extension computed at the center of the diaphragm is within one percent of that computed assuming a spherical shape. The strain ($\varepsilon = \lambda - 1$) is accurate to within six percent in all cases.


ROOM TEMPERATURE RESPONSE OF
ONE MIL STRATOFILM IN FIVE
INCH DIAPHRAGM
COMPUTED STRESS
ASSUMED STRESS
PRESSURE
BIAXIAL STRESS - σ(PSI x 10^-3)
PRESSURE - P(PSI)
2.5
2.0
1.5
1.0
.5
0
0.5
0.5
0.3
0.2
0.1
0
0.5
1.0
1.5
BUBBLE HEIGHT - h_o (INCHES)



STRESS CORRECTION FACTOR
$\kappa = 1.0982 + 0.0322h_o^2$
TEMP
40°C
20°C
0°C
-20°C
-60°C
BUBBLE HEIGHT - h_o (INCHES)
STRESS CORRECTION FACTOR - κ
2.5
2.0
1.5
1.0
0.5
0
1.10
1.15
1.20
1.25

<u>Conclusions</u> - The non-linear deformation of a five inch circular diaphragm has been analyzed by the Ritz minimization method assuming that the material may be characterized as Neo-Hookean in the elastic region. It has been found that:

a. the shape is not spherical;
b. a balanced biaxial stress exists only at the center;
c. this stress may be found by applying a correction factor, κ, which is a function of only bubble height;
d. the strain and extension ratio's are reasonably accurate assuming a spherical shape; and
e. biaxial modulus data on balloon films may be obtainable from the measurement of pressure and bubble height.

Since the Mooney-Rivlin energy density formulation does not adequately characterize polyethylene films with large (beyond yield) deformations, an analysis at high pressures would not be of significance even though it could be accomplished. Therefore, no comment on the state of stress near failure can be made. However, it is apparent that the state of stress is quite complicated and without correction for non-linearities the results of such a test are probably meaningless.